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A possible hidden symmetry and geometrical source of the phase in the CKM matrix

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Abstract. Based on the present data, the three Cabibbo–Kobayashi–Maskawa (CKM) angles may construct a spherical surface triangle whose area automatically provides a "holonomy" phase. By assuming this geometrical phase to be that in the CKM matrix determined by an unknown hidden symmetry, we compare the theoretical prediction on ϵ with experimental data and find the two are consistent within error range. The α, β, γ predicted from this symmetry are also consistent with data. Further applications to the B-physics are briefly discussed. We also suggest restrictions for the Wolfenstein parameters explicitly; the agreement will be tested by more precise measurements in the future.

1 Introduction

Although more than thirty years have elapsed since the discovery of CP violation [1], our understanding about the source of CP violation is still poor. In the Minimal Standard Model (MSM), CP violation is due to the presence of a weak phase in the Cabibbo–Kobayashi–Maskawa (CKM) matrix [2],Cabi. Up to now, all the experimental results are in good agreement with MSM. Nevertheless, the correctness of CKM mechanism is far from being proven. The search for the source of CP violation is a profound and difficult task in high-energy physics [4–8]. In this work we restrict ourselves in the framework of MSM and see if we can find something that was missing in previous studies. First, let us review what we have learned about the CKM matrix.

1.1 Brief review about the CKM theory

Considering all the constraints on the matrix elements, for three-generation quarks, there are three independent angles and a weak phase which cannot be rotated away or absorbed into the quark wavefunctions. The phase, in principle, is independent of the three angles. Much effort has been made to understand the source of the three rotation angles and the phase.

Fritzsch [9,10] noticed that because the eigenstates of the weak interaction are not the quark-mass eigenstates, there should be a unitary transformation to connect the two bases. This would establish a certain relation between the quark masses and the weak interaction mixing angles, while a weak CP phase would be embedded explicitly.

From the general theory of Kobayashi–Maskawa (KM) [2], we know that there can exist a phase factor in the three-generation CKM matrix and that it cannot be removed by redefining the phases of quarks. We can ask, however, whether there is an intrinsic relation between the phase and the three rotation angles.

In Fritzsch's theory, the CKM matrix comes from diagonalizing the U-type and D-type quark-mass matrices, as it is possible that there are certain horizontal relations between different generations of quarks. The proposed symmetry has undergone some modifications for fitting data, especially the top-quark mass; this may hint that there is a broken horizontal symmetry and that the scale of breaking is related to a typical quantity λ.

To look at a concrete example, suppose V_d and V_u diagonalize the mass matrices for D-type and U-type quarks respectively [11]; $V_{\rm KM} \equiv V_u^\dagger V_d$ is the CKM matrix and can be written as

$$V_{\rm KM} = \begin{pmatrix} c_1 & -s_1c_3 & -s_1s_3 \\ s_1c_2 & c_1c_2c_3 - s_2s_3{\rm e}^{{\rm i}\delta} & c_1c_2s_3 + s_2c_3{\rm e}^{{\rm i}\delta} \\ s_1s_2 & c_1s_2c_3 + c_2s_3{\rm e}^{{\rm i}\delta} & c_1s_2s_3 - c_2c_3{\rm e}^{{\rm i}\delta} \end{pmatrix} \quad (1)$$

with the standard notations $s_i = \sin\theta_i$ and $c_i = \cos\theta_i$.

We adopt here the original form of the CKM parametrization. There are some other methods of parametrization, for example Wolfenstein's [12,13,25], and that recommended by the data group [15–17], but it is believed that physics does not change when adopting various parametrizations. Since the original expression (1) has clearer geometrical meaning, we begin our discussion based on this parametrization first, and then will extend our analysis to other parametrization later of this work.

It is well known that the KM parametrization can be viewed as a product of three Eulerian rotation matrices and a phase matrix [11]:

$$V_{\mathrm{KM}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_2 & -s_2 \\ 0 & s_2 & c_2 \end{pmatrix} \begin{pmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -\mathrm{e}^{\mathrm{i}\delta} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_3 & s_3 \\ 0 & -s_3 & c_3 \end{pmatrix}. \tag{2}$$

Some have claimed that the weak CP phase δ, which cannot be eliminated in the three-generation CKM matrix by any means, is introduced artificially, and seems to have nothing to do with the three "rotation" angles. Such an allegation does not seem to be natural.

As equation (2) is widely accepted, we may ask if there exists a hidden symmetry which can relate the weak phase with the three rotation angles.

Based on observation, the recently measured $\theta_1, \theta_2, \theta_3$ satisfy

$$\theta_i + \theta_j \geq \theta_k, \quad i, j, k = 1, 2, 3,$$

and if we take only the positive values of $\sin\theta_i$ as $0 \leq \theta_i \leq \pi/2 (i = 1, 2, 3)$, then

$$\theta_1 + \theta_2 + \theta_3 \leq \frac{3\pi}{2}.$$

Therefore the three angles can construct a spherical surface triangle on a unit sphere in the Hilbert space.

The three angles correspond to three arcs on the unit sphere, and they enclose an area δ. The δ and the three angles have a definite relation

$$\cos\frac{\delta}{2} = \frac{1 + \cos\theta_1 + \cos\theta_2 + \cos\theta_3}{4\cos\frac{\theta_1}{2}\cos\frac{\theta_2}{2}\cos\frac{\theta_3}{2}}. \tag{3}$$

The geometrical meaning of the area is clear. The three vertices A, B and C correspond to $\angle A, \angle B$ and $\angle C$. At each vertex, there are two tangents along the two adjacent arcs defining the positive directions of arcs counterclockwise. If one moves one tangent $\mathbf{t}_1^A$ along the arc AB to vertex B, it comes to $\mathbf{t}_2^B$. If we rotate $\mathbf{t}_2^B$ counterclockwise to $\mathbf{t}_1^B$ by the angle $\pi - \angle B$, let it move to vertex C along arc BC, rotate $\mathbf{t}_2^C$ to $\mathbf{t}_1^C$, and finally move it back to vertex A, then the resultant $\mathbf{t}_2^A$ spans an angle $\pi - \angle A$ with the original vector $\mathbf{t}_1^A$. Geometrically, the three angles $\alpha_1, \alpha_2, \alpha_3$, which transform $\mathbf{t}_2^{A,B,C}$ to $\mathbf{t}_1^{A,B,C}$, respectively, have the relation

$$\begin{aligned} \alpha_1 + \alpha_2 + \alpha_3 &= \pi - \angle A + \pi - \angle B + \pi - \angle C \\ &= 3\pi - (\angle A + \angle B + \angle C) \qquad (4) \\ &= 3\pi - (\pi + \delta) = 2\pi - \delta, \qquad (5) \end{aligned}$$

where δ is exactly that area enclosed by the three arcs and is called the "angular excess". It is noted that for a planar triangle, there is no such angular excess, so the δ-phase is obviously caused by the curved space characteristics, i.e., the affine connection.

Thus δ represents the tangent transformation along the spherical surface triangle, so it can be the variable of a U(1) holonomy transformation group. Naturally, it automatically corresponds to a phase $\mathrm{e}^{\mathrm{i}\delta}$, which is a geometrical phase.

The three Euler angles $\theta_{12}, \theta_{23}, \theta_{31}$ bridge the three-generation quark flavors, and the adopted $\theta_1, \theta_2, \theta_3$ are nothing new but an alternative parametrization scheme, a U(1) phase $\mathrm{e}^{\mathrm{i}\delta}$, could appear in the CKM matrix.

This geometrical phase can be described in another way with an explicit O(3) rotation. As is well known, for a naive O(3) rotation group, a geometric phase can automatically arise while two non-uniaxial successive rotation transformations are being performed [18–23]. For instance, $R_x(\theta_1)$ denotes a clockwise rotation about the x-axis by angle θ_1, while $R_y(\theta_2)$ rotates about the y-axis by θ_2. Supposing a unit-sphere surface, the positive z-axis intersects with the surface at point A. After these two sequential operations $R_y(\theta_2)R_x(\theta_1)$, point A would reach point B via an intermediate point C; by contrast, one can connect A and B by a single rotation $R_{\hat{\xi}}(\theta_3)$, where $R_{\hat{\xi}}(\theta_3)$ denotes a clockwise rotation about the $\hat{\xi}-$axis by θ_3. If one chooses an arbitrary tangent vector $\hat{\alpha}$ at point A, then rotates it to $\hat{\alpha}'$ and $\hat{\alpha}''$ by $R_y(\theta_2)R_x(\theta_1)$ and $R_{\hat{\xi}}(\theta_3)$, respectively, one finds that $\hat{\alpha}'$ does not coincide with $\hat{\alpha}''$, but deviates by an extra rotation. To demonstrate this concretely: If one writes down the rotation in the adjacent representation of $O(3)$, one finds

$$R_{\hat{\eta}}(\delta)R_{\hat{\xi}}(\theta_3) = R_y(\theta_2)R_x(\theta_1), \tag{6}$$

where $R_{\hat{\eta}}(\delta)$ represents a counterclockwise rotation about the $\hat{\eta}-$axis by δ. The phase δ can be a U(1) phase for any non-trivial rotations.

It seems reasonable to assume that the geometrical phase can stand as the CP phase in the CKM matrix and that a hidden symmetry would relate the phase to three mixing angles. We will discuss this in the last section.

Instead of the original form given in (1), the CKM matrix can take the form recommended by the Particle Data Group in [24]

$$V_{\mathrm{KM}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}\mathrm{e}^{-\mathrm{i}\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}\mathrm{e}^{\mathrm{i}\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}\mathrm{e}^{\mathrm{i}\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}\mathrm{e}^{\mathrm{i}\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}\mathrm{e}^{\mathrm{i}\delta_{13}} & c_{23}c_{13} \end{pmatrix} \tag{7}$$

which can be obtained from expression (1) simply by a unitary transformation. Then we have the CP phase δ_{13}, corresponding to δ in (3), as

$$\sin\delta_{13} = \frac{(1+s_{12}+s_{23}+s_{13})\sqrt{1-s_{12}^2-s_{23}^2-s_{13}^2+2s_{12}s_{23}s_{13}}}{(1+s_{12})(1+s_{23})(1+s_{13})}. \tag{8}$$

Since θ_i from (1) correspond to the Euler angles, they have clearer geometrical meaning than θ_{ij} of (7), even though they are exactly equivalent to θ_{ij}. Additionally, though the CP phase in the two expressions takes different values, all physical quantities determined in terms of the two sets of parameters are the same. Later we employ the expression (7), recommended by the Particle Data Group, for the numerical evaluations.

We assume that this geometrical U(1) phase is the weak phase in the CKM matrix, i,e., there are no other underlying physical principles that cause the weak phase in the matrix; only the holonomy phase plays the role of the weak phase. Consequently, we can make deductions which can be tested by comparing phenomenological applications to corresponding experimental data.

1.2 A test from ϵ in $K^0 - \bar{K}^0$ system

So far, the only reliably measured CP violation quantity is ϵ in the K-system, and the mechanism causing $K^0 - \bar{K}^0$ mixing has already been well studied in the framework of MSM. Except for an unknown B_K−factor, one can accurately evaluate ϵ in terms of the CP phase δ as [25,26]

$$|\epsilon| = \frac{G_F^2 m_K f_K^2 B_K M_W^2}{\sqrt{2}(12\pi^2)\Delta m_K} \times[\eta_1 S(x_c) I_{cc} + \eta_2 S(x_t) I_{tt} + 2\eta_3 S(x_c, x_t) I_{ct}] \quad (9)$$

where

$$I_{ij} = \mathrm{Im}(V_{id}^* V_{is} V_{jd}^* V_{js}),$$

and $\eta_1 = 1.38$, $\eta_2 = 0.57$ and $\eta_3 = 0.47$ are the QCD corrections factors [25]. The two functions are

$$S(x) = \frac{x}{4}\left[1 + \frac{3-9x}{(x-1)^2} + \frac{6x^2 \ln x}{(x-1)^3}\right] \quad (10)$$

and

$$S(x,y) = xy\left\{\left[\frac{1}{4} + \frac{3}{2(1-y)} - \frac{3}{4(1-y)^2}\right]\frac{\ln y}{y-x} + \left[\frac{1}{4} + \frac{3}{2(1-x)} - \frac{3}{4(1-x)^2}\right]\frac{\ln x}{x-y} - \frac{3}{4(1-x)(1-y)}\right\}, \quad (11)$$

where $x_c = m_c^2/M_W^2$, $x_t = m_t^2/M_W^2$.

It is understood that the CKM matrix elements evolve under renormalization. All the CKM matrix elements used in the calculations are taken from measurements that are carried out at a lower-energy scale, i.e. $M_K \sim M_B$, so the running effects are not significant, especially for the K-meson system.

The inputs of $| V_{ij} |$ are taken from [24], and we have

$$m_c = 1.5 \text{ GeV}, \quad m_t(m_t^2) = 176 \text{ GeV}, \quad |\epsilon| = 2.3 \times 10^{-3},$$

with all the errors given by the date group. Using (8), we obtain

$$\sin\delta_{13}^{\mathrm{Geo}} \approx 0.967 \sim 0.968,$$

extracting the δ_{13}−value from (9), $\delta_{13}^{\mathrm{exp}}$ becomes

$$\sin\delta_{13}^{\mathrm{exp}} \approx (0.78 \sim 0.99)(\frac{B_{\mathrm{K}}}{0.75}).$$

It is noted that the factor B_K appears for evaluating the hadronic matrix elements. By the vacuum saturation, $B_K = 1$, but other schemes would give quite different values; generally it would fall into a region around 0.7∼ 0.8. Since $|\sin\delta| \le 1$, the measured value of ϵ sets a constraint on the combination of CKM entries and other quantities. Our result of $\sin\delta_{13}^{\mathrm{Geo}}$ does not contradict $\sin\delta_{13}^{\mathrm{exp}}$, according to the present data.

In other words, one can see that considering the experimental error tolerance, the two obtained values are roughly consistent. Since the extraction of δ from the data ϵ still depends on the evaluations of concerned hadronic transition matrix elements which are not reliable so far, (we cannot handle the non-perturbative QCD effects satisfactorily) the deviation between two δ values is reasonable and tolerable.

In any case, this phenomenological application of (8) does not contradict the data. The assumption that the source of the CP phase in the CKM matrix is attributed to the geometrical structure obtains support from this comparison.

1.3 Test of the CKM triangle

The unitarity of the three-generation CKM matrix demands a triangle whose vertex angle values α, β, γ depend on the weak phase. The α, β, γ are defined as

$$\alpha \equiv \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right) \qquad \beta \equiv \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right) \qquad \gamma \equiv \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right). \quad (12)$$

Buras has carefully studied the present data and determined the tolerable range of the parameters as [27]

$$35^\circ \le \alpha \le 115^\circ, \quad 11^\circ \le \beta \le 27^\circ, \quad 41^\circ \le \gamma \le 134^\circ,$$

or, more strictly,

$$70^\circ \le \alpha \le 93^\circ, \quad 19^\circ \le \beta \le 22^\circ, \quad 65^\circ \le \gamma \le 90^\circ.$$

With our simple geometrical ansatz for δ_{13}, we obtain

$$73.2^\circ < \alpha < 94.4^\circ, \quad 10.6^\circ < \beta < 31.3^\circ, \quad 75.0^\circ < \gamma < 75.5^\circ.$$

Obviously, these obtained values of α, β, γ are consistent with those derived by Buras [27]. With the measured V_{ud}, V_{ub}, V_{tb} as inputs, the allowed range of α is narrower, but that of β is broader, than that given in [27]. Especially with this ansatz, the value of γ is very insensitive to the input parameters; it is nearly fixed. This observation can be tested in the future experiments.

1.4 The ranges of Wolfenstein's parameters

As another alternative parametrization, let us have a look at the possible ranges of Wolfenstein's parameters [12]. Provided that the assumption is valid that the δ given in (3) is the weak phase in the CKM matrix, there should be some constraints on Wolfenstein's parameters. In Wolfenstein's parametrization, the CKM matrix reads as

$$V_{\rm W}=\begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho-{\rm i}\eta+{\rm i}\eta\frac{1}{2}\lambda^2) \\ -\lambda & 1-\frac{1}{2}\lambda^2-{\rm i}\eta A^2\lambda^4 & A\lambda^2(1+{\rm i}\eta\lambda^2) \\ A\lambda^3(1-\rho-{\rm i}\eta) & -A\lambda^2 & 1 \end{pmatrix}. \quad (13)$$

Transforming the Kobayashi–Maskawa parameters to Wolfenstein's [11], we have

$$s_1 \approx \lambda, \quad c_1 \approx 1-\frac{\lambda^2}{2} \quad (14)$$

$$s_2 \approx \lambda^2 A[(\rho-1)^2+\eta^2] \quad (15)$$

$$s_3 \approx (\rho^2+\eta^2)^{1/2}A\lambda^2 \quad (16)$$

$$\sin\delta \approx \frac{\eta}{(\rho^2+\eta^2)^{1/2}}\frac{1}{[(\rho-1)^2+\eta^2]^{1/2}}. \quad (17)$$

From (1), we obtain

$$\sin\delta = \Big\{(1+\cos\theta_1+\cos\theta_2+\cos\theta_3) \times\sqrt{\sin^2\theta_1+\sin^2\theta_2+\sin^2\theta_3-2(1-\cos\theta_1\cos\theta_2\cos\theta_3)}\Big\} \Big/\Big\{(1+\cos\theta_1)(1+\cos\theta_2)(1+\cos\theta_3)\Big\}. \quad (18)$$

Substituting (14-16) into (18) and expanding the right-hand side of (18) in powers of λ, with more calculation, we get

$$\sin\delta = \frac{A\sqrt{(1-\rho)^2+\eta^2+(\rho^2+\eta^2)}}{2\sqrt{2}}\lambda^3 + \frac{A[(1-\rho)^2+\eta^2+(\rho^2+\eta^2)-2A^2(1-2\rho)^2]}{2^4\sqrt{2}\sqrt{(1-\rho)^2+\eta^2+(\rho^2+\eta^2)}}\lambda^5 \quad (19)$$

Keeping terms up to λ^3, we identify the right-hand sides of (17) and (20), and we have

$$\frac{\eta}{(\rho^2+\eta^2)^{1/2}[(1-\rho)^2+\eta^2]^{1/2}} \approx \frac{A\sqrt{(1-\rho)^2+\eta^2+(\rho^2+\eta^2)}}{2\sqrt{2}}\lambda^3. \quad (20)$$

(20) sets a constraint on the quark mixing in Wolfenstein's parametrization that is accurate up to order λ^3.

We next give a simple numerical analysis. Setting

$$x=(\rho^2+\eta^2)^{1/2}, \quad (21)$$

$$y=[(1-\rho)^2+\eta^2]^{1/2}, \quad (22)$$

and

$$\eta=\frac{1}{2}\sqrt{2(x^2+y^2)-(x^2-y^2)^2-1}, \quad (23)$$

we derive

$$\frac{\sqrt{2(x^2+y^2)-(x^2-y^2)^2-1}}{xy} = \frac{A\lambda^3}{\sqrt{2}}\sqrt{x^2+y^2}. \quad (24)$$

Fixing $\lambda=0.22$ and $A=0.808\pm0.058$ [26], if we take $y = 0.54\sim1.40$ as input, then $0.22\sim0.46$ for x is permitted. Hence we find that the results are well in agreement with the experimental analysis [8]

$$x=\sqrt{\rho^2+\eta^2}=0.34\pm0.12 \quad (25)$$

and

$$y=\sqrt{(1-\rho)^2+\eta^2}=0.97\pm0.43. \quad (26)$$

It is easy to see that there exist solutions within the experimental error ranges of x and y. Thus the results do not contradict the CKM matrix element measurements.

2 More phenomenological implications

Besides the values of α,β,γ, the simple geometrical ansatz can lead to many phenomenological applications, which will be tested in the B-experiments.

Since this ansatz only refers to the weak phase, the experimentally measured quantities which are sensitive to it are the CP violation effects. The most promising area for measuring CP violation, besides the kaon-system, is the B-system. There are, in general, two different types of CP violation [28], namely, the direct CP violation,

$$A^{\rm dir}_{\rm CP}(B\to f)\equiv\frac{1-|\xi_f|^2}{1+|\xi_f|^2}, \quad (27)$$

and the mixing-induced indirect CP violation,

$$A^{\rm mix-ind}_{\rm CP}(B\to f)\equiv\frac{2{\rm Im}\xi_f}{1+|\xi_f|^2}, \quad (28)$$

where the quantity ξ_f is

$$\xi_f=\exp(-{\rm i}\phi_{\rm M})\frac{A(\bar{B}\to f)}{A(B\to f)}, \quad (29)$$

with $\phi_{\rm M}$ denoting the weak phase in the $B-\bar{B}$ mixing and A is the decay amplitude.

The direct CP violation $A^{\rm dir}_{\rm CP}$ is caused by interference among various channels with different weak and strong phases, while the mixing-induced CP indirect violation $A^{\rm mix-ind}_{\rm CP}$ is determined only by the weak phase. So we choose the observables of $B_s\to\pi^0\phi$ as an example to make a prediction based on the obtained information of the weak CKM phase:

$$\mathcal{A}^{\rm mix-ind}_{\rm CP}(B_s\to\pi^0\phi)=\frac{2(x+\cos\gamma)\sin\gamma}{x^2+2x\cos\gamma+1}, \quad (30)$$

where

$$x\equiv\frac{A_{\rm EW}}{A_{\rm CC}}\approx\frac{\alpha}{2\pi\lambda^2R_b a_2\sin^2\Theta_W}[5B_0(x_t)-2C_0(x_t)] \quad (31)$$

with

$$B_0(x_t) = \frac{1}{4}\left[\frac{x_t}{1-x_t} + \frac{x_t \ln x_t}{(x_t-1)^2}\right], \tag{32}$$

$$C_0(x_t) = \frac{x_t}{8}\left[\frac{x_t-6}{x_t-1} + \frac{3x_t+2}{(x_t-1)^2}\ln x_t\right], \tag{33}$$

$$x_t = \frac{m_t^2}{M_W^2}. \tag{34}$$

Then we obtain

$$\mathcal{A}_{\rm CP}^{\rm mix-ind}(B_s \to \pi^0\phi) \sim -0.57466. \tag{35}$$

There are many other applications that can be tested by the future B-energy experiments. We will stuty these in future works.

3 Conclusion and discussion

In this work, based on an observation of the measured values of the CKM matrix elements and the consideration of a possible hidden symmetry, we have studied the possibility that the weak phase in the CKM matrix has a geometrical basis which relates the weak phase to the three rotation angles; namely, the U(1) geometrical phase emerges due to a hidden symmetry. Even though this is an ad hoc assumption, it is indeed a possibility for the source of the CP phase in the CKM matrix.

Under this assumption, the geometrical phase takes responsibility for the role of the weak phase in the CKM matrix. Obviously, there may be some other mechanisms that bring about the weak phase, and the present data cannot eliminate this possibility. What we show in this work is that the geometrical phase does exist, and can serve as a phase of the CKM matrix. Our numerical results show that the obtained α, β, γ values are consistent with those evaluated in other phenomenological ways. Conversely, the existence of other physical sources of the weak phase cannot be excluded either. The determination of whether or not this geometrical phase can serve as the CKM phase requires more precise experimental measurements.

Moreover, based on this ansatz, we have given constraints to the value ranges of Wolfenstein's parameters, which can be detected by experiments more easily. We have also tried to apply the obtained results to make the prediction $\mathcal{A}_{\rm CP}^{\rm mix-ind}(B_s \to \pi^0\phi)$; future B-factory experiments can verify or negate this simple ansatz.

In conclusion, we claim that the CKM phase may emerge for a geometrical reason; Future experiments are needed to test whether the hypothesis is valid, or other physical sources are responsible for the CKM phase.

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